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A QUEUEING NETWORK APPROACH TO A CREW SCHEDULING PROBLEM.(U)

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A QUEUEING NETWORK APPROACH TO A CREW SCHEDULING PROBLEM

Arnold L. Sweet, Ph.D.

September 1980

Final Report for Period October 1979 - January 1980

USAF SCHOOL OF AEROSPACE MEDICINE
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NOTICES

This final report was submitted by personnel of the Biomathematics Modeling Branch, Data Sciences Division, USAF School of Aerospace Medicine, Aerospace Medical Division, AFSC, Brooks Air Force Base, Texas, under job order 7930-15-04.

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This report has been reviewed by the Office of Public Affairs (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SAM-TR-80-35	2. GOVT ACCESSION NO. AD-A092 834	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A QUEUEING NETWORK APPROACH TO A CREW SCHEDULING PROBLEM.		5. TYPE OF REPORT & PERIOD COVERED Final Report Oct 1979 – Jan 1980
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Arnold L. Sweet, Ph.D.		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS USAF School of Aerospace Medicine (BRM) Aerospace Medical Division (AFSC) Brooks Air Force Base, Texas 78235		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62202F 7930-15-04
11. CONTROLLING OFFICE NAME AND ADDRESS USAF School of Aerospace Medicine (BRM) Aerospace Medical Division (AFSC) Brooks Air Force Base, Texas 78235		12. REPORT DATE September 1980
		13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Queueing networks
Airlift crew-ratio scheduling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The following optimization problem is considered: airplanes fly on prescribed routes and are flown by crews who must be rested after the passage of a certain interval of flying time. Rested crews who are stationed at bases on the routes can be deployed to keep the planes in flight. For a prescribed number of planes and crews, it is desired to place the crews at bases such that the amount of time spent by the planes in the air is a maximum. This paper investigates the use of a queueing network model as a means of formulating the optimization problem. Upper and lower bounds are presented for the expected

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20. ABSTRACT (Continued)

fraction of time planes are airborne. These bounds may be useful in solving the optimization problem by means of a simulation model.

A QUEUEING NETWORK APPROACH TO A CREW SCHEDULING PROBLEM

STATEMENT OF PROBLEM

Airplanes fly on prescribed routes and are flown by crews who must be rested after the passage of a certain interval of flying time. Rested crews who are stationed at bases on the routes can be deployed to keep the planes in flight. For a prescribed number of planes and crews, it is desired to place the crews at bases in such a manner that the amount of time spent by planes in flight, measured over any fixed interval of time, is a maximum.

This problem has previously been approached by simulation modeling. The objective of this paper is to provide an analytical model which hopefully will aid in the interpretation of results obtained using the simulation model.

APPROACH TO PROBLEM

The model to be presented here is a queueing network model. Such models have been successful in providing a robust description of computer network operation (2) and manufacturing systems (4) and are currently being used to model transportation systems (5).

The system process will be represented by a state vector, and transitions between states will obey the Chapman-Kolmogoroff equations (1). Thus, the process is a Markov process and is also a birth and death process. Solution of the resulting birth and death equations will yield indicators of system performance.

MODEL 1: ONE PLANE ON ONE LOOP

As an example of the methods to be employed, consider one airplane flying on a route such that the plane always returns to the base from which it began. This route will be called a loop. The route consists of bases, at which the plane must stop, and of flights between the bases. It is assumed that the flight times from takeoff at base i to landing at base i+1, denoted by X_i, are independent random variables (RVs) with expectation $1/\alpha_i$. Due to the requirement that a crew must be rested after a specified interval of flying time, upon arrival at a base a plane is considered to be in one of two conditions: ready (R), or not ready ($\bar{R}$). It is ready if the flight time has been less than some specified length, k_i, and not ready otherwise. If ready, the crew which was on the plane will return to the plane when it takes off again, and the amount of time the plane spends at base i is denoted by a RV $S_i^{(2)}$, with expectation $1/\mu_{i2}$. If the plane is not ready, the crew must rest at the base before the plane can leave again. In this case, the time the plane spends at base i is denoted by a RV $S_i^{(1)}$, with expectation $1/\mu_{i1}$, where $\mu_{i1}<\mu_{i2}$. The base times are assumed to be independent for different bases, but the base times at base i are dependent on the length of the

flight time from base i-1. The probability of arriving at base i+1 in a state $\overline{R}$ will be denoted by $r_{i,i+1}$, where

$$r_{i,i+1} = P(\lambda_i > k_i) \qquad (1)$$

The effect of staging a crew at a base i is to allow a nonready plane to remain at the base a time $S_i^{(2)}$, instead of a time $S_i^{(1)}$. Thus it can be seen that the location of the plane and its state of readiness will suffice as state variables, as the utilization of the crews can be derived from a knowledge of the plane's states as a function of time.

Consider a loop with M bases, and let the plane begin at base one. Let there be no extra crews for staging. The expected time to return to base one is the sum of the expected times between bases, plus the expected time at bases, and is given by

$$L = \sum_{i=1}^{M-1} [\alpha_i^{-1} + r_{i,i+1}\mu_{i+1,1}^{-1} + (1-r_{i,i+1})\mu_{i+1,2}^{-1}] + \alpha_M^{-1} . \qquad (2)$$

The expected fraction of time the plane is in flight is given by

$$F = L^{-1} \sum_{i=1}^{M} \alpha_i^{-1} . \qquad (3)$$

Consider the staging of one crew at a base. It is desired to choose the base at which to place the crew such that F is a maximum. This can be accomplished by minimizing

$$H = \sum_{i=1}^{M-1} K_i , \qquad (4)$$

where

$$K_i = r_{i,i+1} (\mu_{i+1,1}^{-1} - \mu_{i+1,2}^{-1}) \qquad i=1,2,\ldots,M-1. \qquad (5)$$

Thus, the crew should be staged at the base which has the largest value of K_i, and it can be seen that K_i is the expected savings in base time for one stop of a plane when a crew is staged at base i.

If there are C crews available for staging, $C \leq M-1$, then F is maximized by placing the crews at the C bases with the C largest values of K_i.

To formulate the problem using Markov process theory, assume that the flight times and base times have exponential distributions. The state of the system is the location of the plane, and, if the plane is at a base, it is also necessary to specify whether the plane arrived in a state of readiness. We will now assume that the plane can circle the loop indefinitely, and that we are interested in the steady-state behavior of the system. The number of possible states is 3M. A simple example will suffice to illustrate the method of solution. Consider the two-base loop in Figure 1, where each base or flight is identified as a node in a network. The nodes are labeled with the reciprocals of the expected values of the RVs associated with each node. Figure 2 shows the state transition diagram for the network in Figure 1, where each state is identified by a numeral corresponding to the node in Figure 1. The steady-state birth and death equations are

$$\begin{aligned}
\mu_{11}P_1 &= \alpha_2 r_{21} P_6 \\
\mu_{12}P_2 &= \alpha_2(1-r_{21})P_6 \\
\alpha_1 P_3 &= \mu_{12}P_2 + \mu_{11}P_1 \\
\mu_{21}P_4 &= \alpha_1 r_{12} P_3 \\
\mu_{22}P_5 &= \alpha_1(1-r_{12})P_3 \\
\alpha_2 P_6 &= \mu_{21}P_4 + \mu_{22}P_5
\end{aligned} \tag{6}$$

where P_i is the probability that the system is in state i. The system of equations 6 is redundant, so any one of them can be eliminated, and a unique solution is obtained by using the condition that the probabilities sum to one. The solution is

$$\begin{aligned}
P_1 &= r_{21}\mu_{11}^{-1}L^{-1} \\
P_2 &= (1-r_{21})\mu_{12}^{-1}L^{-1} \\
P_3 &= \alpha_1^{-1}L^{-1} \\
P_4 &= r_{12}\mu_{21}^{-1}L^{-1} \\
P_5 &= (1-r_{12})\mu_{22}^{-1}L^{-1} \\
P_6 &= \alpha_2^{-1}L^{-1}
\end{aligned} \tag{7}$$

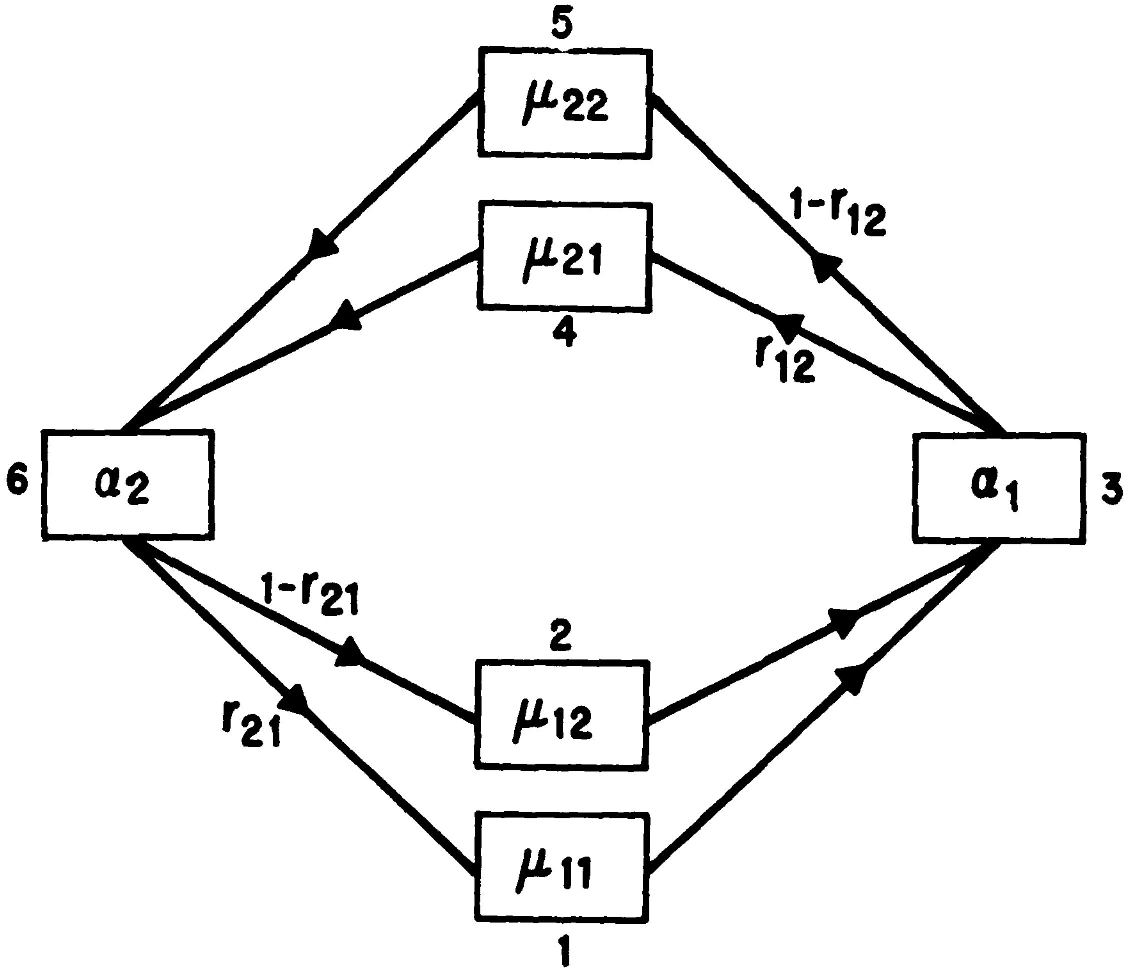

Figure 1. Network representation of a two-base loop.

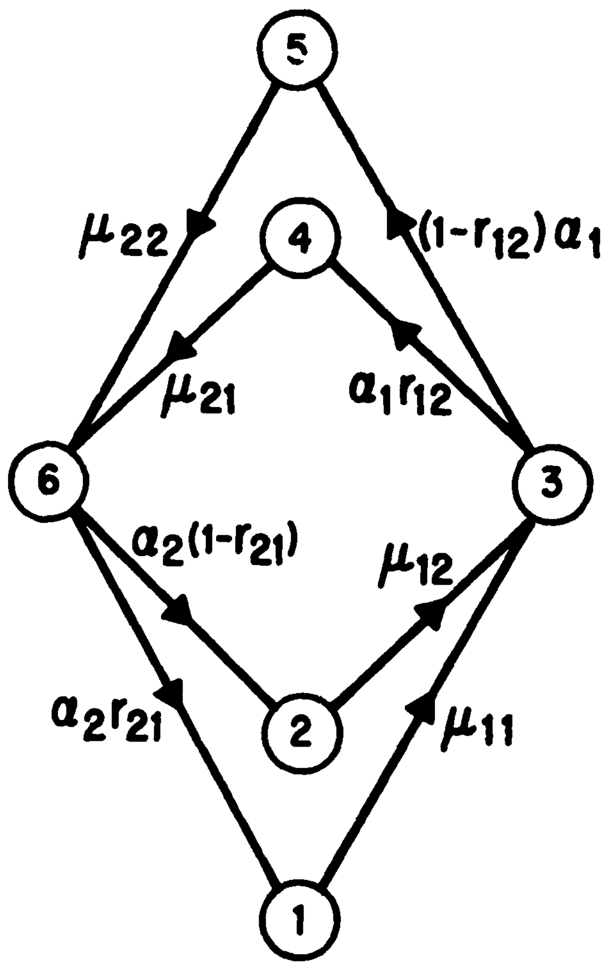

Figure 2. State-transition diagram for the network in Figure 1.

where

$$L = \alpha_1^{-1} + \alpha_2^{-1} + \mu_{12}^{-1} + \mu_{22}^{-1} + r_{21}(\mu_{11}^{-1} - \mu_{12}^{-1}) + r_{12}(\mu_{21}^{-1} - \mu_{22}^{-1}), \qquad (8)$$

and the expected fraction of time spent in flight is

$$F = P_3 + P_6 = (\alpha_1^{-1} + \alpha_2^{-1})L^{-1} . \qquad (9)$$

Comparison of equations 9 and 3 shows that, again, a staging crew would be placed at the base with the largest value of K_i in order to maximize F.

Hereafter, in examining more complex models, attention will be focused on the problem of maximizing the expected fraction of time spent in flight under steady-state conditions. It is hoped that the optimal solution of the steady-state problem would be of help in finding an optimal solution for a finite interval of time using a simulation model. We also note the robust character of the solution (equation 9), which depends only on expected times and the r's.

MODEL 2: TWO LOOPS WITH A SHARED BASE

Consider two planes which leave and return to a common base. The planes may have different return times. It is assumed that the return times are independent and exponentially distributed, with expectations α_1^{-1} and α_2^{-1} for planes one and two, respectively. (See Figure 3.)

If no staging crews are present at the base, then each plane is stochastically independent. Using the method of birth and death equations, it can be shown that, if $F_i(C)$ is the expected fraction of time plane i is in flight when C crews are available for staging, then

$$F_1(0) + F_2(0) = \sum_{i=1}^{2} \frac{\alpha_i^{-1}}{\alpha_i^{-1} + \mu_2^{-1} + r_{i3}(\mu_1^{-1} - \mu_2^{-1})} \quad . \qquad (10)$$

It is also true that if two crews are available for staging, then again the two planes are stochastically independent, since any plane in a state of nonreadiness can use a fresh crew, and

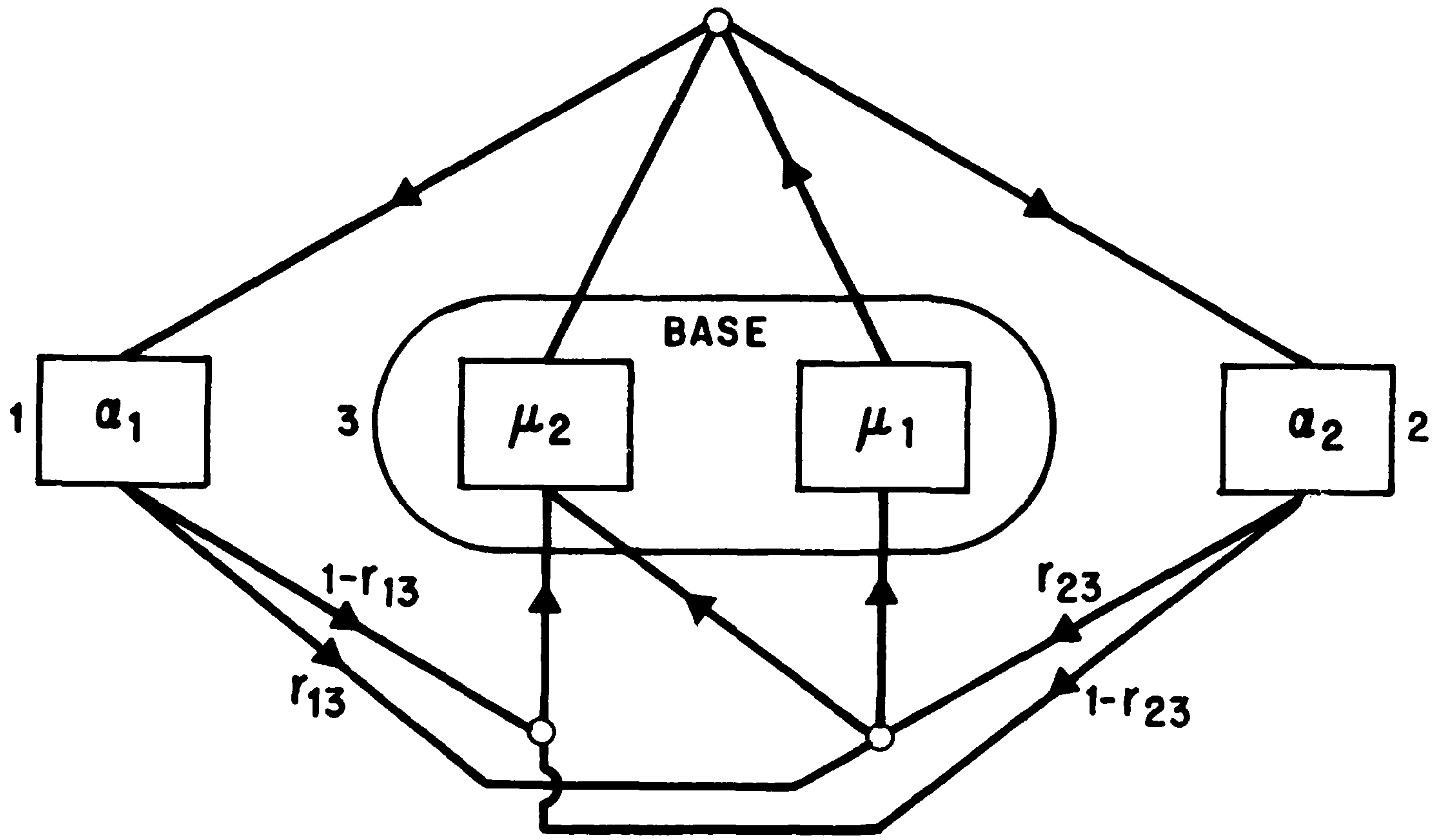

Figure 3. Two-loop, one-base network.

$$F_1(2) + F_2(2) = \sum_{i=1}^{2} \frac{\alpha_i^{-1}}{\alpha_i^{-1} + \mu_2^{-1}} . \qquad (11)$$

When one crew is available for staging, the following queue discipline will be used:

a. The base time for a plane arriving in state R will be $S^{(2)}$.

b. The base time for a plane arriving in state $\overline{R}$, and finding no other plane at the base will be $S^{(2)}$.

c. The base time for a plane arriving in state $\overline{R}$, and finding a plane at the base which arrived in state R will be $S^{(2)}$.

d. The base time for a plane arriving in state $\overline{R}$ and finding a plane at the base which arrived in state $\overline{R}$ will be $S^{(1)}$.

The above queue discipline causes the planes to be stochastically dependent. The state of the system is defined by each plane's location. When a plane is at the base, it is also necessary to denote whether the plane is R or $\overline{R}$. If both planes are $\overline{R}$ it is necessary to denote which plane arrived first (f) or second ($\overline{f}$). There are 10 states, and they are described in Table 1. Figure 4 shows the state-transition diagram, and the associated equations are

$$
\begin{aligned}
(\alpha_1 + \alpha_2)P_1 &= \mu_2(P_2 + P_3 + P_5 + P_8) + \mu_1(P_7 + P_{10}) \\
(\alpha_2 + \mu_2)P_2 &= \alpha_1(1-r_{13})P_1 + \mu_2P_4 \\
(\alpha_1 + \mu_2)P_3 &= \alpha_1(1-r_{23})P_1 + \mu_2P_4 \\
2\mu_2P_4 &= \alpha_2P_2 + \alpha_1P_3 + \alpha_2(1-r_{23})P_5 + \alpha_1(1-r_{13})P_8 \\
(\alpha_2 + \mu_2)P_5 &= \alpha_1r_{13}P_1 + \mu_1P_6 \qquad (12) \\
(\mu_1 + \mu_2)P_6 &= \alpha_2r_{23}P_5 + \alpha_1P_7 \\
(\alpha_1 + \mu_1)P_7 &= \mu_2P_6 \\
(\alpha_1 + \mu_2)P_8 &= \alpha_2r_{23}P_1 + \mu_1P_9 \\
(\mu_1 + \mu_2)P_9 &= \alpha_1r_{13}P_8 + \alpha_2P_{10} \\
(\alpha_2 + \mu_1)P_{10} &= \mu_2P_9 \ .
\end{aligned}
$$

The expected fractional flight times are

$$F_1(1) = P_1 + P_3 + P_7 + P_8$$

$$\text{and} \tag{13}$$

$$F_2(1) = P_1 + P_2 + P_5 + P_{10} \cdot$$

TABLE 1. STATE ENUMERATION FOR THE TWO-LOOP, ONE-BASE, ONE STAGING CREW MODEL

	State Description	
State Number	Plane 1	Plane 2
1	Flight	Flight
2	Base, R	Flight
3	Flight	Base, R
4	Base, R	Base, R
5	Base, $\bar{R}$, f	Flight
6	Base, $\bar{R}$, f	Base, $\bar{R}$, $\bar{f}$
7	Flight	Base, $\bar{R}$, $\bar{f}$
8	Flight	Base, $\bar{R}$, f
9	Base, $\bar{R}$, $\bar{f}$	Base, $\bar{R}$, f
10	Base, $\bar{R}$, $\bar{f}$	Flight

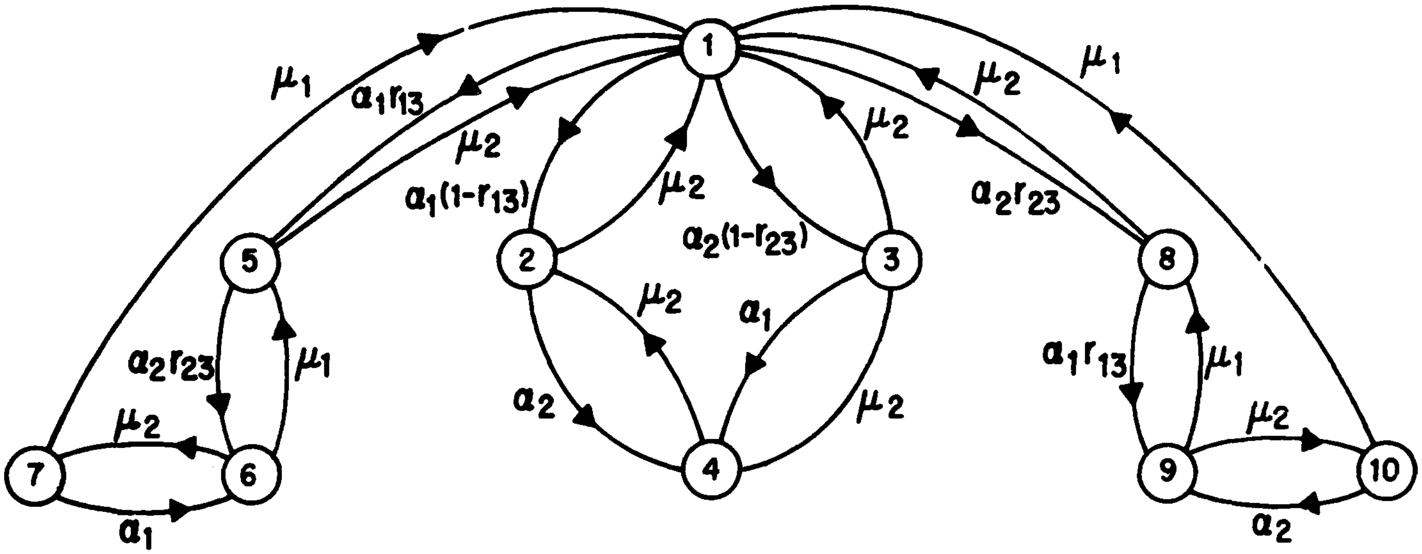

FIGURE 4. TWO-LOOP, ONE-BASE, ONE-STAGING CREW STATE-TRANSITION DIAGRAM.

Let W_i be the expected time (waiting time) plane i spends at the common base. Then, viewing the motion of plane i as an alternating renewal process (3), it can be shown that

$$F_i(1) = \frac{\alpha_i^{-1}}{\alpha_i^{-1} + W_i} \qquad i=1,2 . \qquad (14)$$

Equations 12 were solved by numerical methods, and some typical results are shown in Table 2. Since $F_i(C)$ is the expected value of a RV which is 1 when plane i is in flight, and zero otherwise, the correlation coefficient ρ between these RVs for the two loops can be computed from the solution to the birth and death equations. It is of interest to note that ρ is very small, and that the addition of a second staging crew increases fractional flight time very little. The reason for this is that when one crew is available for staging, the probability that two planes are at the base, each in state $\overline{R}$, is small (.00067). Thus, occasions when another staging crew would be useful are rare.

TABLE 2. RESULTS FOR TWO-LOOP, ONE-BASE MODEL, WHEN α_1=.063, α_2=.125, μ_1=.083, μ_2=2, r_{13}=.2, and r_{23}=.5

Staging Crews (C)	$F_1(C)$	$F_2(C)$	W_1	W_2	ρ
0	.8496	.5605	2.8096	6.2741	0
1	.9656	.9375	.5656	.5331	.0028
2	.9695	.9412	.5000	.5000	0

Various computations showed that as the probability of R decreased, the variation in values of $F_i(C)$ as C increased became smaller.

A GENERAL MODEL AND APPROXIMATIONS

It can now be seen that a network of planes, loops, and bases can be modeled using birth and death equations. These equations are linear in the unknown probabilities, and can be solved using numerical techniques. Once the probabilities are obtained, the expected fraction of flight time can be computed for each plane. The difficulty with this approach is that the

number of states increases roughly as three times the number of bases times the number of planes. In addition, it is extremely tedious to derive the birth and death equations. As noted previously, other network problems have proved amenable to such an approach. The reason that this has been so is that the solution of the birth and death equations for those could be computed as a product of probabilities with reference only to the network representation, and thus it was not necessary to construct the state-transition diagrams (2). This fortunate set of circumstances also applies to the single-loop model, but not to the two-loop model. The reason that a product solution is not applicable in the two-loop model is that it is necessary to keep the planes distinguishable when they are both in state $\overline{R}$ at a base.

Despite the impracticality of the use of the birth and death equations to solve the optimization problem for a large number of states, the above development can be used to find an approximate solution. Consider the placing of one staging crew in a network. Recall that if the number of staging crews is zero, the planes are stochastically independent. Consider N planes (each considered to be flying its own loop). Let the i^{th} loop have M_i bases (some of which may serve many loops). Let T_i be the sum of the expected flight times in loop i, B_i the sum of the expected waiting times at all bases when in state R, and H_i the sum of the expected savings in base time at every base in loop i (see equation 4). Then the expected fraction of flight time for all planes is (see equations 8 and 9)

$$F(0) = \sum_{i=1}^{N} \frac{T_i}{T_i + B_i + H_i} . \tag{15}$$

For any particular loop, a staging crew should be placed at the base where K is the greatest. If this base has no other loops in common, $F_i(1)$ can be computed. On the other hand, if the base is common, the effect of placing the staging crew there cannot be computed without solving the birth and death equations. If an approximation to H_i could be formed for common bases, one would then choose the base which maximized F(1). Then the next staging crew could be added using the same procedure, and this would continue until all available crews were used. A derivation for an upper bound for H_i follows.

Consider a base where N planes can land, and where C staging crews have been placed, $0<C<N$. Choose a particular plane, called plane 1, and let $r_i, i=1,2,\ldots,N$ be the probability that plane i arrives at the base in state $\overline{R}$. Let A be the event "plane 1 arrives at the base to find at least C planes there, plane 1 arrives in state $\overline{R}$, and at least C of the planes found at the base by plane 1 are in state $\overline{R}$." The expected time spent at the base by plane 1, W, is

$$W = P(\overline{A})\mu_2^{-1} + P(A)\mu_1^{-1} = \mu_2^{-1} + P(A)(\mu_1^{-1} - \mu_2^{-1}) . \tag{16}$$

An upper bound for W, and hence a lower bound for F_i (C), can be found by noting that

$$P(A) \leq P \text{ (plane 1 arrives in state } \overline{R}\text{, and at least C planes found at the base by plane 1 are in state } \overline{R}\text{).} \tag{17}$$

The right-hand side of equation 17, denoted by P_u, can be computed using the independence of the flight times. Thus, for example, if N=3 and C=1,

$$P_u = r_1[r_2(1-r_3) + r_3(1-r_2) + r_2r_3] \text{ .} \tag{18}$$

Thus, an approximate optimization procedure could be based on the maximization of

$$F(C) = \sum_{i=1}^{N} \frac{T_i}{T_i + B_i + U_i} \qquad 0<C<N \tag{19}$$

where U_i is an upper bound for H_i in equation 15, and is computed using the appropriate P_u times $(\mu_1^{-1} - \mu_2^{-1})$ for the base under consideration. As an illustration, consider the network of Figure 5. There are two bases on loop one, and one of the bases can be used by both planes. There are twenty-two states in the transition diagram (which is not shown). If C=0, the loops are independent. If C=1, the crew can be staged at either base. If C=2, both crews can be staged at the common base (making the loops independent again), or one crew can be staged at each base. Three crews is the most that can be usefully deployed. Note that when C=N, an upper bound for F(C) is obtained.

Table 3 shows results which can be used to compare the optimization procedure using the lower bound with the procedure based on the solution to the birth and death equations. It can be seen that if C=1, the use of the lower bound leads to deployment at the common base, which is the correct choice. Likewise, if C=2, one crew at each base is the optimal deployment using the approximate solution, which is again the correct solution. Also shown in Table 3 is the correlation, ρ, between F_1 and F_2.

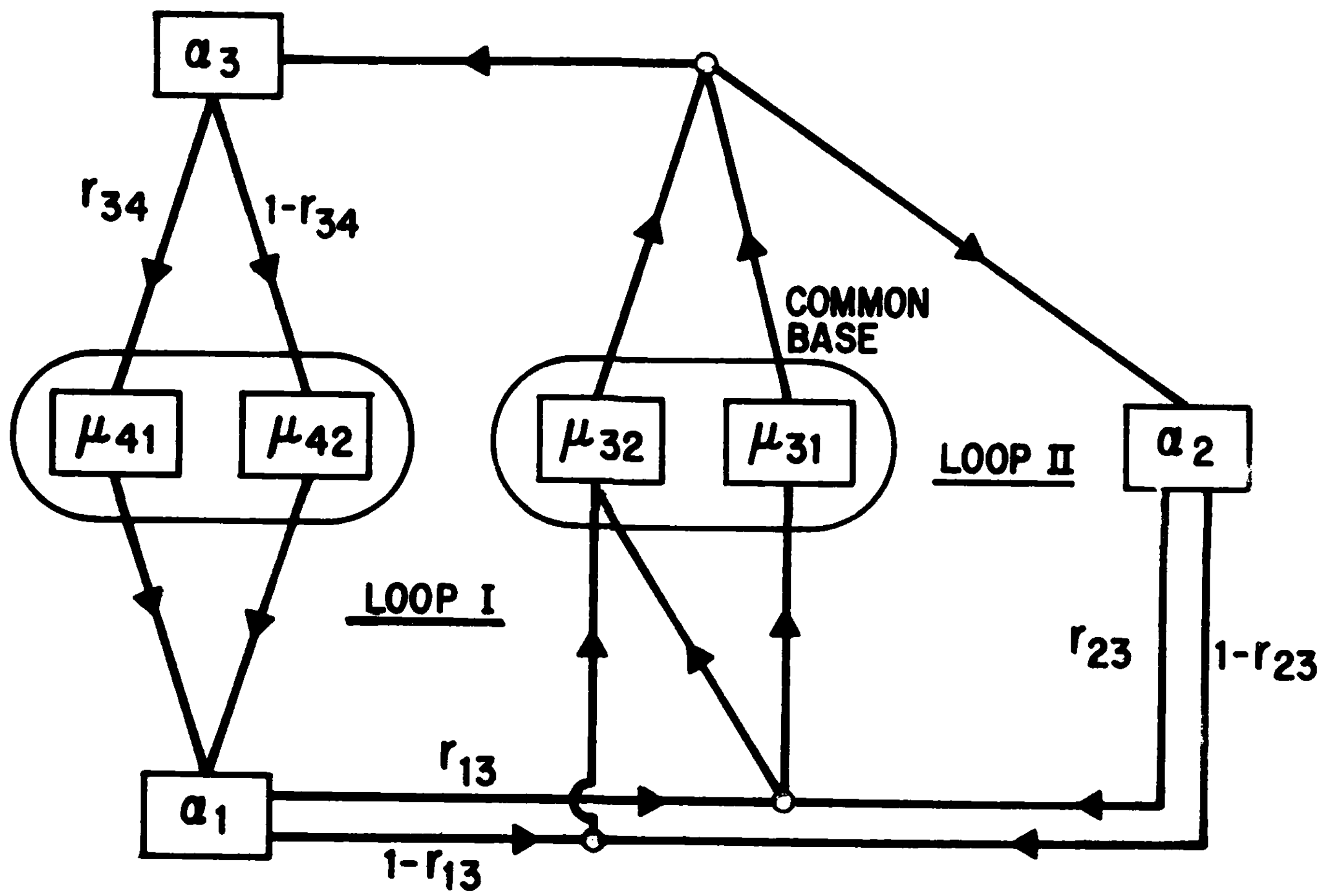

Figure 5. Two-loop, two-base network.

TABLE 3. TWO-LOOP, TWO-BASE MODEL. $\alpha_1=\alpha_2=\alpha_3=.063$, $\mu_{31}=\mu_{41}=.083$, $\mu_{32}=\mu_{42}=2.0$, $r_{13}=r_{23}=r_{34}=.10$.

Number of Staging Crews, C, and Deployment	Exact Solution			Lower Bound	
	$F_1(C)$	$F_2(C)$	ρ	$F_1(C)$	$F_2(C)$
0	.9056	.9056	0	N.A.	
1 @ common base	.9363	.9694	.00007	.9333	.9627
1 @ noncommon base	.9364	.9056	0	N.A.	
2 @ common base	.9364	.9695	0	N.A.	
1 @ each base	.9694	.9694	.00013	.9661	.9627
3	.9695	.9695	0	N.A.	

Table 4 shows results for a situation where deployment at the noncommon base when C=1 is optimal. In a situation where the optimal choice is not clear, the loss in making an incorrect decision would be small.

TABLE 4. TWO-LOOP, TWO-BASE MODEL. $\alpha_1=\alpha_3=.168$, $\mu_2=.063$, $\mu_{31}=\mu_{41}=.083$, $\mu_{32}=\mu_{42}=2.0$, $r_{13}=r_{23}=.10$, $r_{34}=.50$

Number of Staging Crews, C, and Deployment	Exact Solution			Lower Bound	
	$F_1(C)$	$F_2(C)$	ρ	$F_1(C)$	$F_2(C)$
0	.6002	.9056	0	N.A.	
1 @ common base	.6372	.9693	-.00009	.6334	.9627
1 @ noncommon base	.8467	.9056	0	N.A.	
2 @ common base	.6373	.9695	0	N.A.	
1 @ each base	.9223	.9692	.00020	.9143	.9627
3	.9225	.9695	0	N.A.	

Examination of Tables 3 and 4 shows that very little is gained from placing a second crew at the common base, as the upper bound for F is almost attained with one crew at each base. Note also that the correlations between F_1 and F_2 are very small.

SUMMARY AND COMMENTS

It has been shown how the increase in the expected fraction of time planes spend in flight is related to a decrease in the expected waiting times spent at bases which are visited. These waiting times are proportional to the difference in the expected waiting time when a staging crew is and is not available. The constant of proportionality is a function of the number of staging crews, the number of planes, and the probability that the crews on

arriving planes are allowed to continue to fly the planes that they arrived on. An upper bound for the constant of proportionality was found, and thus upper and lower bounds for the expected fraction of flight time could be formed. This enables a computation to be made of the deployment of crews in order to maximize the expected fraction of flight time for all planes. Hopefully, this deployment could be used to guide the placing of crews in a simulation model.

Exact solutions for some simple networks indicated that as the number of staging crews increased, the gain in expected flight time became progressively smaller. This indicates that the use of simulation to optimize such networks may not give accurate results as the number of staging crews are increased, because the effect of the sampling errors may be such that they are greater than the magnitude of the gain in expected flight time.

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Notation

B_i = sum of expected base times when in state R, for loop i.

C = number of staging crews available for deployment.

F = expected fraction of time a plane is in flight.

F(C) = expected fraction of time all planes are in flight when C staging crews are used.

$F_i(C)$ = same as F(C), but for plane i only.

H = sum of expected savings in base times (eqn. (4)).

H_i = same as H, but only for loop i.

k_i = limiting value for a crew's flight time from base i to base i+1.

K_i = expected savings in base time if a crew is deployed at base i (eqn. (5)).

L = expected loop time.

M_i = number of bases on loop i.

N = number of planes.

P_i = state probability.

P_u = upper bound for P(A).

P(A) = probability that an arriving plane will spend a time $S^{(1)}$ at a base.

$r_{i,i+1}$ = $P(X_i > k_i)$.

R = a crew's state of readiness to fly.

$S_i^{(1)}$ = time at base i for a plane not in state R.

$S_i^{(2)}$ = time at base i for a plane in state R.

T_i = sum of expected flight times for loop i.

U_i = upper bound for H_i.

W = expected waiting time (base time)

X_i = flight time from base i to base i+1.

$\bar{\alpha_i}^{1} = E(X_i)$

$\bar{\mu}_{i\,1}^{1} = E\ (S_i^{(1)})$

$\bar{\mu}_{i\,2}^{1} = E\ (S_i^{(2)})$

ρ = correlation coefficient for flight times.

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